Does Monte Carlo Analysis Actually Overstate Tail Risk In Retirement Projections?

7/5/2017

Executive Summary

The most common criticism of using Monte Carlo analysis for retirement planning projections is that it may not fully account for occasional bouts of extreme market volatility, and that it understates the risk of "fat tails" that can derail a retirement plan. As a result, many advisors still advocate using rolling historical time periods to evaluate the health of a prospective retirement plan, or rely on actual-historical-return-based research like safe withdrawal rates, or simply eschew Monte Carlo analysis altogether and project conservative straight-line returns instead.

In this guest post, Derek Tharp – our Research Associate at Kitces.com, and a Ph.D. candidate in the financial planning program at Kansas State University – analyzes Monte Carlo projection scenarios relative to actual historical scenarios, to compare which does a better job of evaluating sequence of return risk and the potential for an "unexpected" bear market… and finds that in reality, Monte Carlo projections of a long-term retirement plan using typical return and standard deviation assumptions are actually far more extreme than real-world historical market scenarios have ever been!

For instance, when comparing a Monte Carlo analysis of 10,000 scenarios based on historical 60/40 annual return parameters to historical returns, it turns out that 6.5% of Monte Carlo scenarios are actually worse than even the worst case historical scenario has ever been! Or viewed another way, a 93.5% probability of success in Monte Carlo is actually akin to a 100% success rate using actual historical scenarios! And if the advisor assumes lower-return assumptions instead, given today's high market valuation and low yields, a whopping 50% to 82% of Monte Carlo scenarios were worse than any actual historically-bad sequence has ever been! As a result, despite the common criticism that Monte Carlo understates the risk of fat tails and volatility relative to using rolling historical scenarios, the reality seems to be the opposite – that Monte Carlo projections show more long-term volatility, resulting in faster and more catastrophic failures (to the downside), and more excess wealth in good scenarios (to the upside)!

So how is it that Monte Carlo analysis overstates long-term volatility when all criticism has been to the contrary (that it understates fat tails)? The gap emerges because of a difference in time horizons. When looking at daily or weekly or monthly data – the kind that leveraged investors like hedge funds often rely upon – market returns do exhibit fat tails and substantial short-term momentum effects. However, in the long run – e.g., when looking at annual data – not only do the fat tails entirely disappear, but long-term volatility actually has a lack of any tails at all! The reason is that in the long-run, returns seem to exhibit "negative serial correlation" (i.e., mean reversion – whereby longer-term periods of low performance are followed by periods of higher performance, and vice-versa). Yet by default, Monte Carlo analysis assumes each year is entirely independent, and that the risk of a bear market decline is exactly the same from one year to the next, regardless of whether the market was up or down for the past 1, 3, or 5 years already. In other words, Monte Carlo analysis (as typically implemented in financial planning software) doesn't recognize that bear markets are typically followed by bull markets (as stocks get cheaper and eventually rally), and this failure to account for long-term mean reversion ends out projecting the tails of long-term returns to be more volatile than they have ever actually been!

> *The bottom line, though, is simply to recognize that despite the common criticism that Monte Carlo analysis and normal distributions understate "fat tails", when it comes to long-term retirement projections, Monte Carlo analysis actually overstates the risk of extreme drawdowns relative to the actual historical record – yielding a material number of projections that are worse (or better) than any sequence that has actually occurred in history. On the one hand, this suggests that Monte Carlo analysis is actually a more conservative way of projecting the safety of a retirement plan than "just" relying on rolling historical returns. Yet on the other hand, it may actually lead prospective retirees to wait too long to retire (and/or spend less than they really can), by overstating the actual risk of long-term volatility and sequence of return risk!*

(Michael's Note: This post was written by Derek Tharp, our new Research Associate at Kitces.com. In addition to his work on this site, Derek is finishing up his Ph.D. in the Personal Financial Planning program at Kansas State University, and assists clients through his RIA Conscious Capital. Derek is a Certified Financial Planner, and can be reached at derek@kitces.com.)

Monte Carlo Analysis And Relying On A Normal Distribution

While already included in most financial planning software solutions, Monte Carlo analysis remains a somewhat controversial projection tool for financial planners, due to the fact that it commonly relies on a normal distribution to project the probability of future returns in a world where many have suggested that returns are not actually normally distributed. Which raises the question of whether or to what extent Monte Carlo analysis projections might be understating the risk of a retirement plan?

To evaluate this question, it helps to first revisit what the normal distribution is, and how it actually works.

The normal (or Gaussian) distribution is one of the most commonly used distributions in both natural and social sciences. From a statistical perspective, there are many neat and useful characteristics of a normal distribution, including the fact that distributions of many naturally occurring phenomena—such as human height, IQ scores, and measurement errors—can be approximated with a normal distribution.

One of the most common ways advisors use normal distributions is in Monte Carlo simulations. A Monte Carlo simulation models future outcomes by randomly selecting returns, based on the likelihood that they occur – where the "likelihood" is quantified by the average and standard deviation of a normal distribution, which recognizes that extreme returns (to the upside or downside) are less common.

For instance, if an advisor wants to conduct a simulation assuming a 5% average return and 10% standard deviation, the following normal distribution would be produced:

SAMPLE OF A NORMAL (GAUSSIAN) DISTRIBUTION OF MARKET RETURNS

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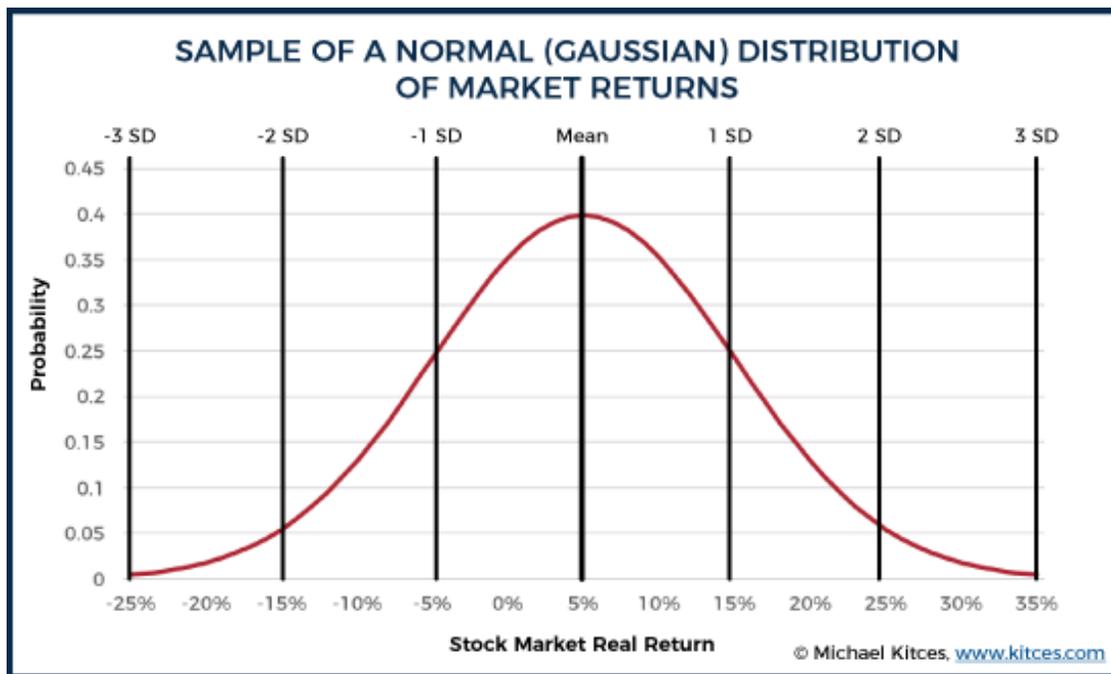
As you can see in the graphic above, the most common return that would be selected randomly is 5%, while more extreme values would come up less frequently. Specifically, in a normal distribution, 68% of the values occur within 1 standard deviation of the mean (in this case, a return between -5% and +15%), 95% of the results are within 2 standard deviations (e.g., -15% to 25%), and 99.7% of the values fall within 3 standard deviations (which would be returns from -25% to 35%). In other words, given the parameters above, returns greater than 35% or less than -25% (i.e., more extreme than 3 standard deviations), would only be expected to occur about 0.3% of the time.

However, as noted earlier, a leading criticism of Monte Carlo analyses is that "extreme" returns can occur more often than the 0.3% frequency implied by a normal distribution – in other words, the "tails" of the distribution are "fatter" (i.e., more frequent) than what a normal distribution would project, particularly to the downside (i.e., a catastrophic bear market). Although, notably, the extent to which market returns have "fat tails" depends on the time horizon involved. While many studies have shown that daily and monthly stock returns appear to have fatter tails, when projected annually (as is common in financial planning projections), the downside fat tails largely disappear.

This lack of "fat tails" in long-term annual stock returns also holds true for 60/40 portfolio returns, based on the large-cap U.S. stocks and Treasury Bills. Using Robert Shiller's data going back to 1871, we can use a Shapiro-Wilk test to examine whether annual returns exhibit a statistically significant deviation from a normal distribution – and the findings suggest they do not. In other words, while there may be "fat tails" in the short-term (daily or monthly) return data, it averages out by the end of the year.

Comparing Historical Returns And Monte Carlo Projections

Notwithstanding the lack of fat tails evident in the data for annual-return retirement projections, many financial planners eschew Monte Carlo analysis and prefer to simulate prospective retirement plans using *actual* historical return sequences, which, by definition, capture actual real-world volatility (potential fat tails and all) that has occurred in the past. In fact, the entire origin of Bengen's "4% rule" safe withdrawal rate was simply to model retirement spending through rolling historical time periods, identify the worst historical scenario that has ever occurred, and use that as a baseline for setting a "safe" initial spending rate in retirement.

We can get a sense of whether or to what extent Monte Carlo analysis understates long-term tail risk relative to actual historical returns by actually comparing them in side-by-side retirement projections.

Data available from Robert Shiller, going back to 1871, provides a sequence of actual historical returns (for large-

cap U.S. stocks, and Treasury Bills) that we can use to evaluate the impact of "real-world" market volatility on retirement, and then compare it to what a Monte Carlo analysis projection would show using the same average return and volatility as that historical data.

If we assume a 60/40 portfolio of stocks and fixed income (annually rebalanced), and a portfolio that begins at $1,000,000 (to make the numbers round and easy), the starting point is to look back into the initial withdrawal rate that would have "worked" for any particular rolling 30-year period (based on actual historical return sequences). Accordingly, we find that in the worst-case scenario the "safe" spending rate was $40,766 at the beginning of the first year (with spending adjusted each subsequent year for inflation). This equates to a 4.08% initial withdrawal rate (relative to the starting account balance), reaffirming Bengen's 4% rule.

Notably, though, *most* of the time a 4.08% initial withdrawal rate is unnecessary. If we assume that the retiree *always* takes that $40,766 of initial spending and adjusts each subsequent year for inflation, we end up with the following range of wealth outcomes.
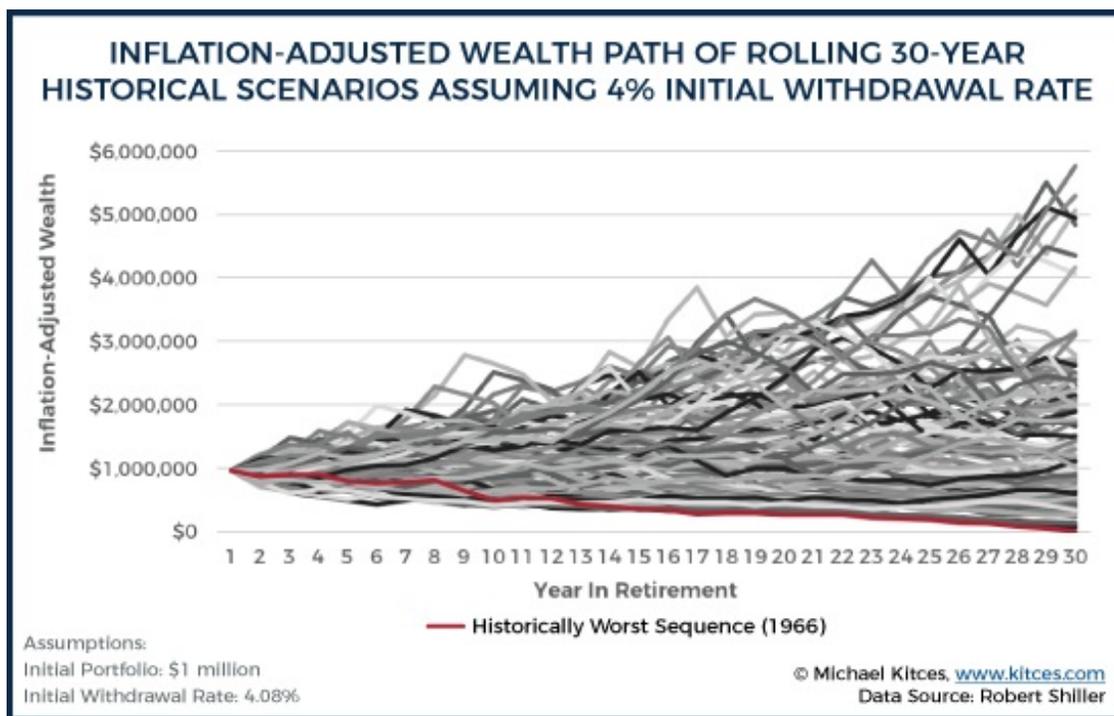
In the chart above, the worst 30-year sequence in history (beginning in 1966) is indicated in red. For that one worst-case scenario, the retiree still makes it to the end (but just barely), thus necessitating that 4.08% initial withdrawal rate. In all the other scenarios, though, the 4.08% safe withdrawal rate is actually "too" conservative, and the portfolio finishes with sometimes *very* substantial (inflation-adjusted) wealth left over at the end.

The 4% safe withdrawal rate is often 'too' conservative, leaving substantial wealth at the end!
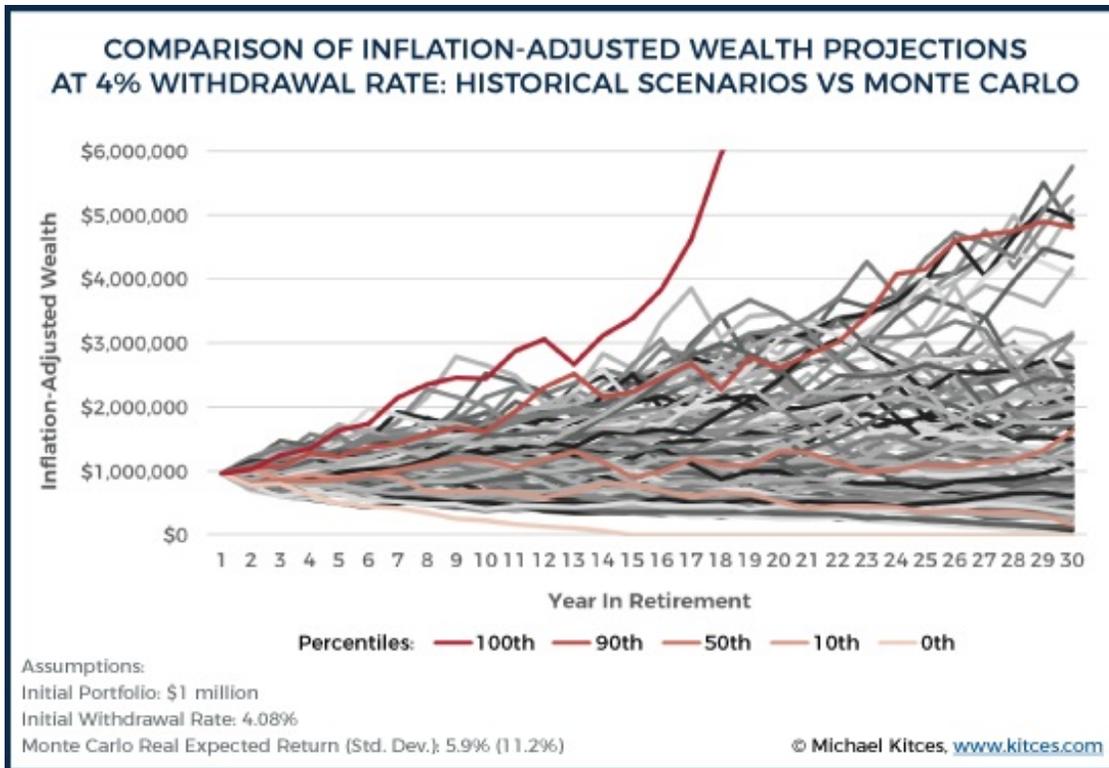
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In fact, the retiree actually dies with *more* inflation-adjusted wealth than they had entering retirement in over 50% of scenarios by using that 4% initial withdrawal rate! And in 30% of scenarios, retirees would have died with nearly 200% of their initial inflation-adjusted wealth or more! In nominal terms, this means a retiree who entered retirement with $1,000,000 would have died with $4,000,000 (or more) 30% of the time, even *after* taking an inflation-adjusted annual distribution each year (assuming an average inflation rate of 2.5%).

Given these results based on the actual historical record, we can then re-evaluate the same retirement time period, using the same historical return data, by running a Monte Carlo analysis that uses the average return and standard deviation of the annual returns that underlie this historical time period. In this case, the data from 1871 to 2015 show

that the annually rebalanced 60/40 portfolio had an average annual *real* return of 5.9%, with a standard deviation of 11.2%.

Using these real return and standard deviation inputs, the chart below shows various percentiles outcomes of a Monte Carlo analysis with 10,000 iterations. To see how "bad" the worst Monte Carlo scenarios are at a 4.08% initial withdrawal rate, these percentile outcomes are compared to the actual historical return sequences (which includes the historical worst-case scenario in 1966, which necessitated the 4.08% initial withdrawal rate in the first place).
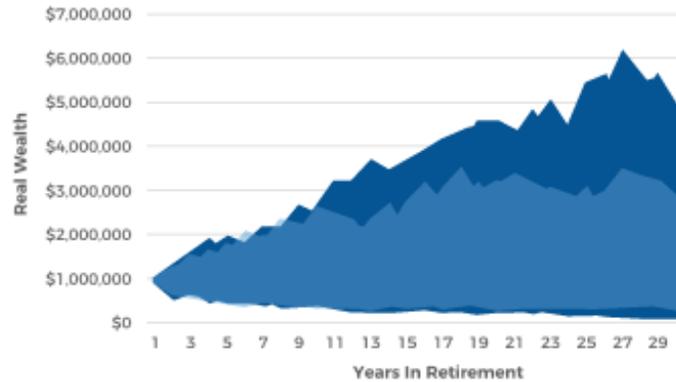

COMPARISON OF INFLATION-ADJUSTED WEALTH PROJECTIONS AT 4% WITHDRAWAL RATE: HISTORICAL SCENARIOS VS MONTE CARLO

Assumptions:
Initial Portfolio: $1 million
Initial Withdrawal Rate: 4.08%
Monte Carlo Real Expected Return (Std. Dev.): 5.9% (11.2%)

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As the chart reveals, the best and worst Monte Carlo scenarios (0th and 100th percentiles) were actually *far more extreme* than any actual historical best or worst scenario. With the Monte Carlo analysis, the worst-case retiree scenario ran out of money as early as only 15 years into retirement, even though the same spending rate *never* actually ran out in any of the 114 rolling 30-year historical scenarios. Conversely, under the best Monte Carlo scenario, the retiree died with nearly $27 million in real wealth, even though the best case historical scenario finished with "just" $6 million of inflation-adjusted wealth at the end. The following chart summarizes the ending real wealth values at various percentiles.

Monte Carlo results are often far more extreme than actual historical scenarios!

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MONTE CARLO VS HISTORICAL SCENARIOS TERMINAL WEALTH AT DIFFERENT PERCENTILES

Percentile	MC Based on Historical Parameters	Historical Scenarios
0th	Depleted in Year 15	$1
10th	$165,000	$365,000
25th	$695,000	$600,000
50th	$1,620,000	$1,100,000
75th	$2,970,000	$2,080,000
90th	$4,810,000	$2,775,000
100th	$26,750,000	$5,755,000

Assumptions:
Initial Portfolio: $1 million
Initial Withdrawal Rate: 4.08%
Monte Carlo Real Expected Return (Std. Dev.): 5.9% (11.2%)

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Data Source: Robert Shiller

The key point is that, despite the common criticism that Monte Carlo *understates* volatility relative to using rolling historical scenarios, the reality, at least relative to historical scenarios, is actually the opposite – the Monte Carlo projections show *more* long-term volatility, resulting in faster and more catastrophic failures (to the downside), and more excess wealth in good scenarios (to the upside). In fact, the 0th percentile historical scenario (worst case scenario, running out of money at the end of 30 years) was actually the 6.5th percentile in Monte Carlo, while the 50th percentile historically (finishing with $1.1M remaining) was the 37th percentile in the Monte Carlo projection, and the 100th percentile (best case scenario) historically was "just" the 93.7th percentile result with the Monte Carlo projection.

Of course, it shouldn't be surprising that Monte Carlo comes up with more extreme best and worst-case scenarios, given that it's based on 10,000 trials (while the historical data has only 114 series of overlapping data points), and a higher volume of random trials should yield results that are more extreme.

Nonetheless, the point remains: plans that succeed in 100% of historical scenarios may only be projected to succeed 95% of the time in Monte Carlo, suggesting that the problem may not be that Monte Carlo is *understating* risk, but that it is being *overstated* given the frequency that Monte Carlo projects scenarios that are more extreme than those that have been experienced in the past! (As it wasn't just a 1-in-10,000 Monte Carlo scenario that was more extreme than the historical record… instead, it was 650-in-10,000!)

The Gap Between Monte Carlo And Reality: Long Sequences Without Mean Reversion

To understand why there is such a gap between the Monte Carlo results and the actual historical scenarios, we can delve further by looking at the *actual* sequence of real returns that underlies each.

First, let's examine the initial 15 years of *real* returns of a 60/40 portfolio starting in 1966 (the worst retirement starting year of the last century):

ANNUAL RETURNS OF FIRST 15 YEARS OF WORST HISTORICAL SEQUENCE

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	13	15
Annual Real Return	-5.0%	7.9%	4.2%	-7.6%	2.4%	7.1%	8.5%	-14.6%	-17.9%	18.1%	3.5%	-9.5%	3.0%	0.2%	6.9%
Compound Growth Rate From Start	-5.0%	1.2%	2.2%	-0.4%	0.2%	1.3%	2.3%	0.0%	-2.1%	-0.3%	0.0%	-0.8%	-0.5%	-0.5%	0.0%
Purchasing Power	95.0%	102.4%	106.7%	98.6%	101.0%	108.1%	117.3%	100.2%	82.3%	97.2%	100.5%	90.9%	93.6%	93.8%	100.4%

Assumption: Monte Carlo Real Expected Return (Std. Dev.): 5.9% (11.2%) © Michael Kitces, www.kitces.com

As the chart reveals, this worst-case historical 30-year sequence didn't get off to a great start (as you'd expect, given the impact of sequence of returns). After 10 years, the average annual compound growth rate of the portfolio was negative. Which means, after accounting for inflation (and without even considering taxes or fees), a retiree with a 60/40 portfolio had already gone *backward* in inflation-adjusted terms. And this is *before* considering the impact of distributions themselves (i.e., these are time-weighted returns, not dollar-weighted). At the end of the entire 15 years, returns had *still* gone nowhere in real terms, and the portfolio was merely treading (inflation-adjusted) water.

By contrast, though, the sequence starting in 1966 was a walk in the park compared to the worst Monte Carlo scenario:

ANNUAL RETURNS OF FIRST 15 YEARS OF WORST MONTE CARLO SEQUENCE

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	13	15
Annual Real Return	-14.7%	6.9%	-15.0%	-15.6%	0.6%	1.9%	-9.7%	-11.4%	0.4%	-4.3%	2.7%	2.4%	-0.1%	-19.0%	-2.4%
Compound Growth Rate From Start	-14.7%	-4.5%	-8.1%	-10.1%	-8.0%	-6.4%	-6.9%	-7.5%	-6.6%	-6.4%	-5.6%	-5.0%	-4.6%	-5.7%	-5.5%
Purchasing Power	85.3%	91.2%	77.5%	65.4%	65.8%	67.0%	60.6%	53.7%	53.9%	51.6%	53.0%	54.3%	54.2%	44.0%	42.9%

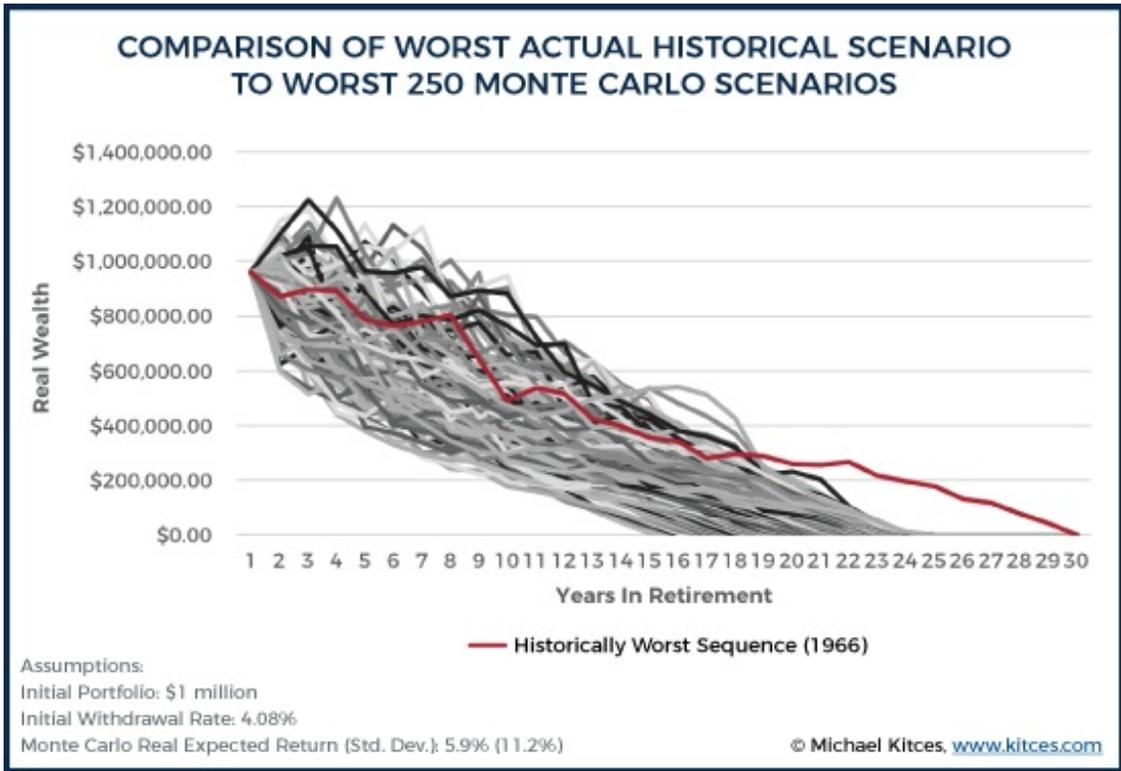Assumption: Monte Carlo Real Expected Return (Std. Dev.): 5.9% (11.2%) © Michael Kitces, www.kitces.com

0% real returns start to sound good after looking at how extreme the worst-case Monte Carlo results were! 10 years in, the retiree has experienced a -6.6% compounded real return, which, in annualized form, somewhat masks how bad these returns truly are. In real dollars, this retiree's portfolio would have lost roughly 50% of its purchasing power in the first decade, and by year 15, it would only buy roughly 43% of its original value! And that's *before* the impact of the withdrawals themselves!

Notably, the extremes within the year-to-year *annual* real returns of the two scenarios are roughly comparable. The historical sequence included 14.6% and 17.9% declines, while the Monte Carlo scenario included 19.0% and 15.6% declines. What differs, though, is the sequencing and frequency of negative returns – specifically, the way in which the Monte Carlo scenario *continues* to string together *multiple* bear markets without *ever* having a recovery.

In the historical scenario, the 2-year decline in years 8 and 9 (which represents the 1973-74 bear market) was followed by an 18.1% rebound. In the Monte Carlo scenario, though, the portfolio started out with a substantial market decline of 14.7%, had "just" a 6.9% rebound, then experienced a 1973-74 style decline in years 3 and 4… after which there was a less-than-3% rebound, followed by *another* 2-year bear market. And then, after mostly flat returns for 5 years, there was *another* bear market. And thus, by the end, the Monte Carlo scenario eroded a whopping 57% of its real purchasing power, even though the *worst* actual historical scenario *merely broke even* on purchasing power after 15 years (which admittedly is still a *horrible* sequence compared to a long-term real return that averages closer to 5%/year!).

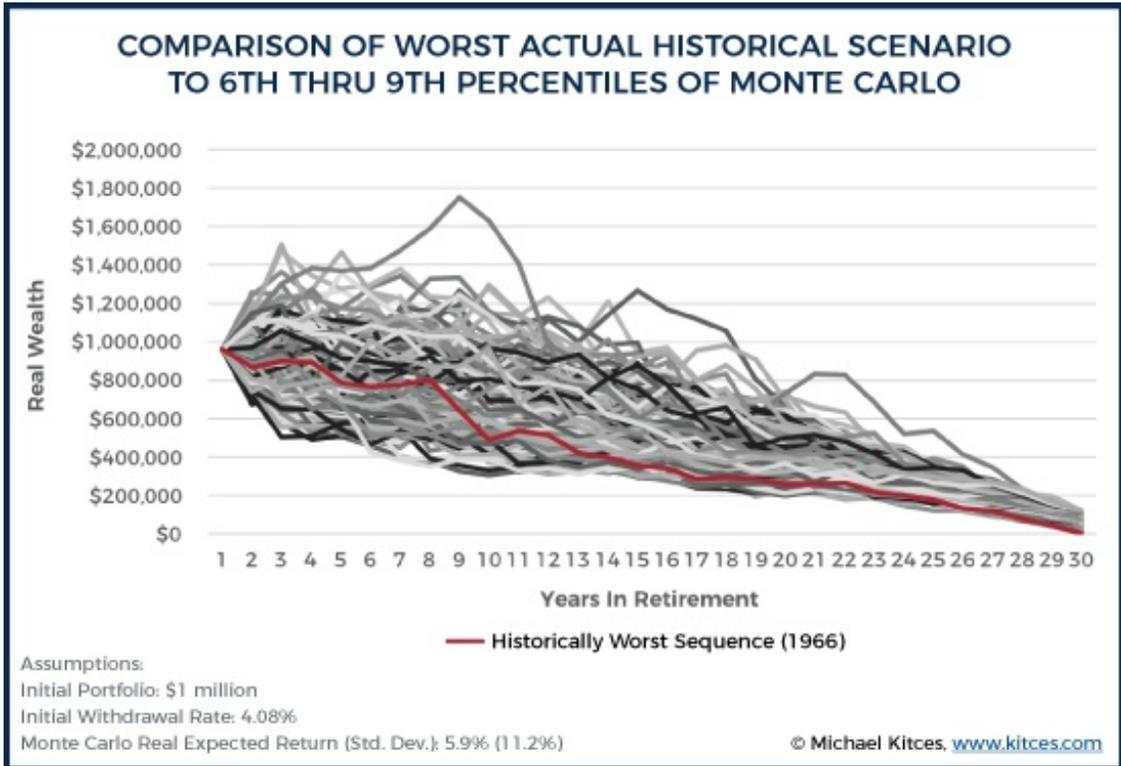And notably, this phenomenon isn't unique to just the worst-case Monte Carlo scenario. If we look at the worst 250 Monte Carlo scenarios (i.e., the worst 2.5% of the 10,000 trials), *all* of them end up with an ongoing series of extreme negative returns with little-or-no rebounds, such that they all deplete in 15-24 years. Yet again, the actual worst-case *historical* scenario with this spending rate still lasted for 30 years.

COMPARISON OF WORST ACTUAL HISTORICAL SCENARIO
TO WORST 250 MONTE CARLO SCENARIOS

Assumptions:
Initial Portfolio: $1 million
Initial Withdrawal Rate: 4.08%
Monte Carlo Real Expected Return (Std. Dev.): 5.9% (11.2%)

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It's not until we move up to the worst 650 to 900 Monte Carlo scenarios (i.e., the 6.5[th] to 9th percentiles) that we even find Monte Carlo results "as bad" as the actual historical record. Or, viewed another way, a 93.5% probability of success in Monte Carlo was the equivalent of a 100% success rate in the historical record!

A 93.5% Monte Carlo success rate was equivalent to a 100% success rate based on historical data!

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COMPARISON OF WORST ACTUAL HISTORICAL SCENARIO
TO 6TH THRU 9TH PERCENTILES OF MONTE CARLO

Assumptions:
Initial Portfolio: $1 million
Initial Withdrawal Rate: 4.08%
Monte Carlo Real Expected Return (Std. Dev.): 5.9% (11.2%)

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Still, even in these Monte Carlo trials, the fact that Monte Carlo analysis does not consider the natural rebound

effects of markets remains evident. For instance, consider the standout iteration above that jets off by itself to the upside, approaching $1.8 million in year 8 above. In this case, the scenario started off with astonishingly strong returns for a 60/40 portfolio, as *real* returns were 21%, 20%, 9%, 2%, 4%, 9%, 10%, 13% over the first 8 years, which is an average annual compound *real* return of 10.8% for 8 years! Yet this scenario still finishes in the bottom 10% overall, because the next series of returns were equally improbable in the other direction, with the sequence of -5%, -11%, -27%, -25%, -17%, 18%, -15%, -4%, -14%, over the next 9 years, which equates to a -11.9% *compounded real return for 9 years…* or a 68% *real* decline in 9 years, far worse than even the worst actual 9-year sequence in history!

In other words, the gap that is emerging between Monte Carlo and historical market returns may not *just* be due to the fact that 10,000 Monte Carlo scenarios produce the potential for more extreme market declines than just 114 actual 30-year rolling historical scenarios. Instead, another distinction may be that with actual market returns, markets tend to at least pull back after several years of strong returns and to *rebound* after a crash. Yet, in the most extreme Monte Carlo projections, they often just keep rising or declining in dramatic fashion, regardless of how expensive or cheap the stocks are getting.

Tail Risk, Momentum, and Mean Reversion In Stock Market Returns

Mathematically, Monte Carlo analysis assumes that each year's returns are entirely independent of the prior year(s). In other words, whether the prior year was flat, saw a slight increase, or a raging bull market, Monte Carlo analysis assumes that the odds of a bear market decline the following year are exactly the same. And the odds of a subsequent decline in the following years also remains exactly the same, regardless of whether it would be the first or eighth consecutive year of a decline!
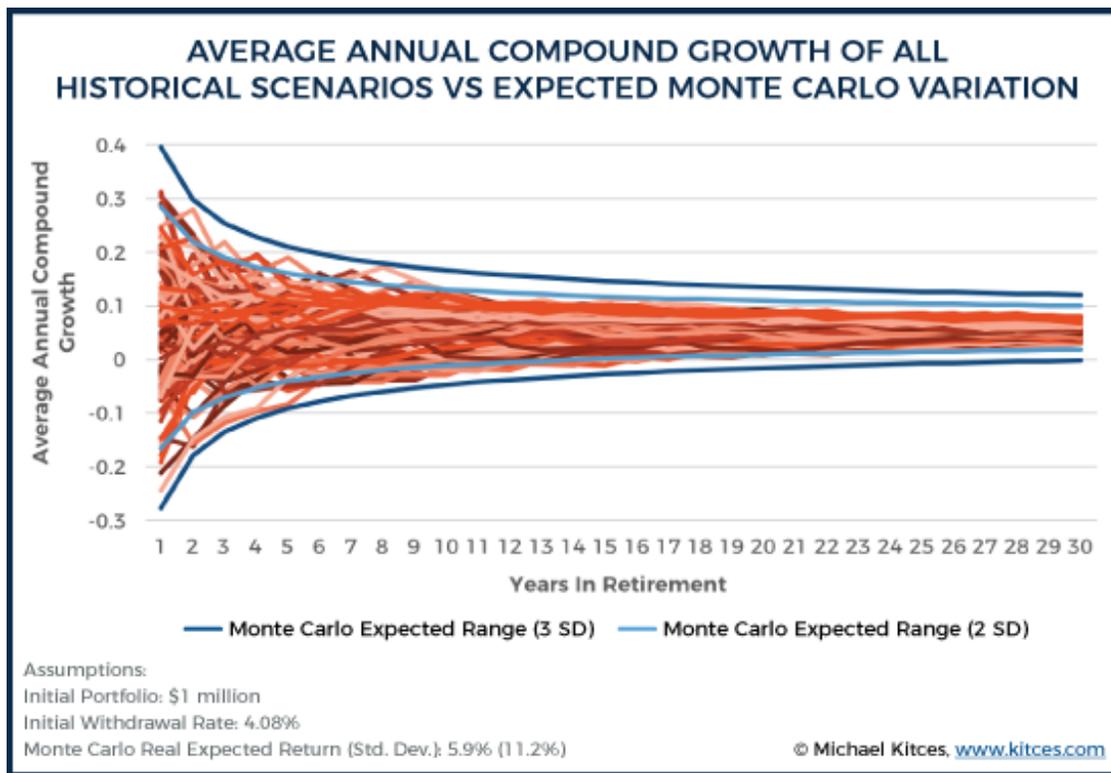
Yet, a look at real-world market data reveals that this isn't really the case. Instead, market returns seem to exhibit at least two different trends. In the short-run, returns seem to exhibit "positive serial correlation" (i.e., momentum – whereby short-term positive returns are more likely to be followed by positive returns, and vice-versa), and, in the long-run, returns seem to exhibit "negative serial correlation" (i.e., mean reversion – whereby longer-term periods of low performance are followed by periods of higher performance, and vice-versa).

Because Monte Carlo projections are long-term projections spanning multiple years (or decades), it is the "negative serial correlation" (i.e., mean reversion) which may cause the "tails" of Monte Carlo projections to actually be *more* volatile and extreme than anything in the historical record. In other words, because most Monte Carlo analyses don't account for mean reversion, this specific aspect of Monte Carlo projections will actually tend to *overstate* tail risk (not understate it!).

Because Monte Carlo analyses don't account for mean reversion, tail risk can be OVERstated!!

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For instance, the chart below shows the average annual compound growth rates (in real returns) of all scenarios in the historical record, and compares these to what Monte Carlo analysis would *predict* at the 2 standard deviation and 3 standard deviation levels of compound returns, simply using an (annually independent) normal distribution based on the historical mean and standard deviation. If fat tails were present in the returns of a 60/40 portfolio, the most extreme historical records should fall outside of the 2 standard deviation Monte Carlo bands about 5 percent of the time; yet, as the results appear to reveal, over 30-year time periods, historical markets have *never* fallen outside the 2 standard deviation thresholds, indicating long-term returns may actually exhibit *less* volatility than Monte Carlo analysis would predict! Which, in turn, means that Monte Carlo analysis may actually *overstate* the tail risk in long-term returns!

AVERAGE ANNUAL COMPOUND GROWTH OF ALL
HISTORICAL SCENARIOS VS EXPECTED MONTE CARLO VARIATION

Monte Carlo Expected Range (3 SD) — Monte Carlo Expected Range (2 SD)

Assumptions:
Initial Portfolio: $1 million
Initial Withdrawal Rate: 4.08%
Monte Carlo Real Expected Return (Std. Dev.): 5.9% (11.2%)
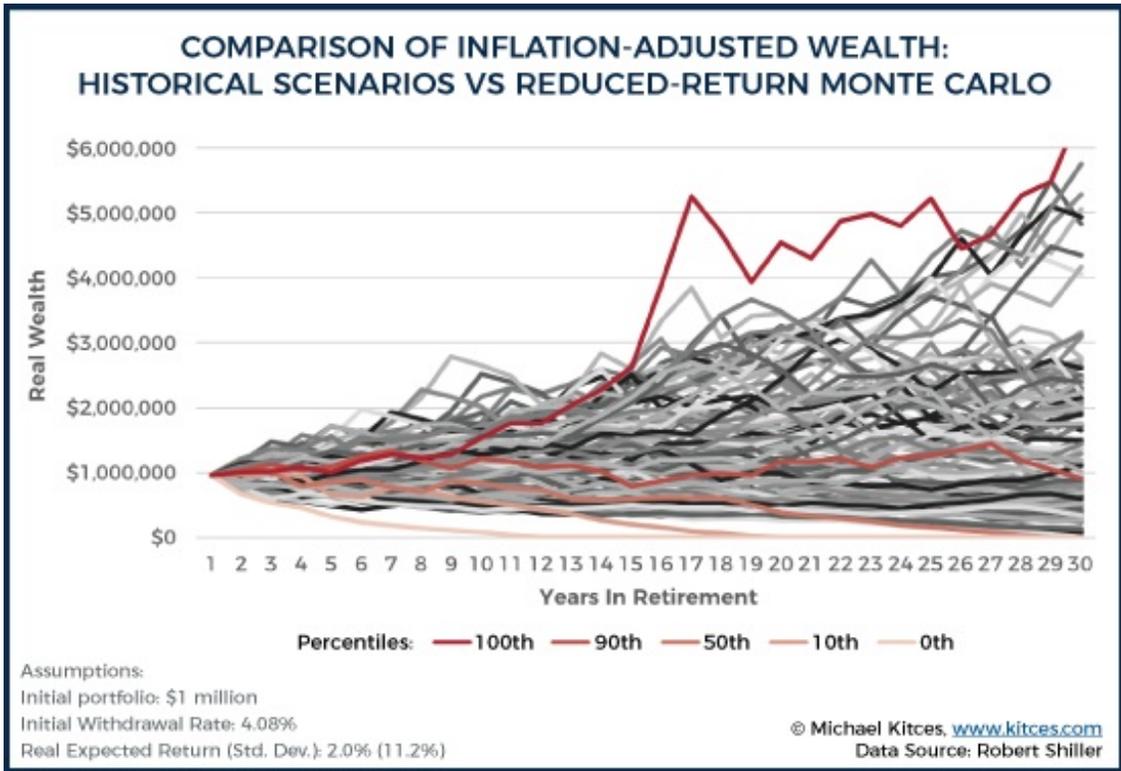
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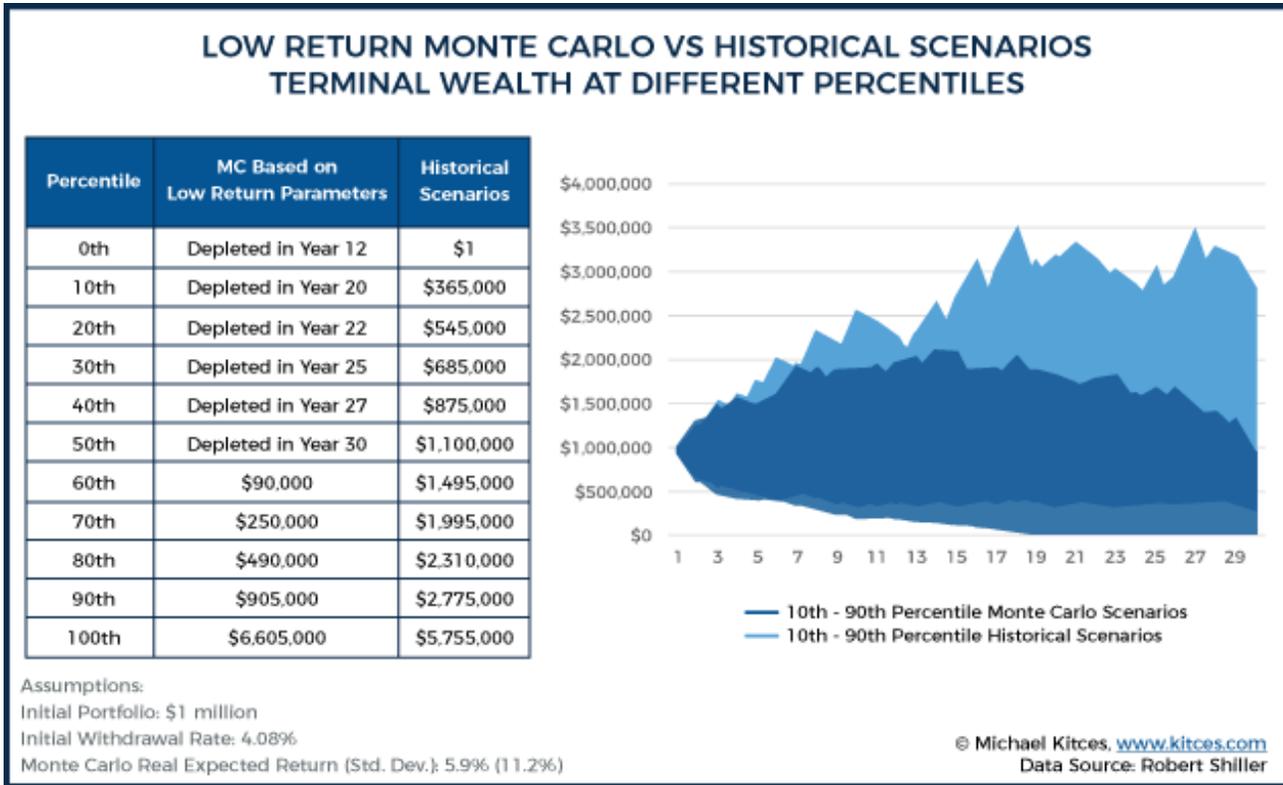## Do Monte Carlo Retirement Projections Overstate Or Understate Tail Risk?

In the case of highly leveraged hedge funds, the reality is that it doesn't *matter* if returns are mean-reverting in the long run, because a margin call can end a highly leveraged investor in a single day or week (as occurred in the famous case of Long-Term Capital Management). However, financial planning projections typically do rely on *annual* data (with annual withdrawals), not daily (or weekly or monthly) data. And, in the annual context, these kinds of extreme events are actually just a short-term distraction that vanish in long-term data! Instead, the problem is actually that Monte Carlo analysis may be *overstating* the dangers of failure by *overestimating* the frequency of long-term tail risk (in addition to overstating the potential upside in good scenarios)!

And this distinction is especially important given the popular tendency of financial advisors to reduce long-term return assumptions as a way of adjusting for Monte Carlo's perceived understatement of tail risk. Recent academic research, such as the 2017 article Planning for a More Expensive Retirement by David Blanchett, Michael Finke, and Wade Pfau, urges that it is important to consider the planning consequences of forward-looking real portfolio returns in the 0% to 2% range (compared to a historical average of 5.9% for a 60/40 portfolio).

To explore the potential impact of these reduced return assumptions, we can evaluate 10,000 new Monte Carlo simulations using the same standard deviation (11.2%) but a lower mean real return (2%), and then compare the Monte Carlo results to actual historical scenarios.

COMPARISON OF INFLATION-ADJUSTED WEALTH:
HISTORICAL SCENARIOS VS REDUCED-RETURN MONTE CARLO

Assumptions:
Initial portfolio: $1 million
Initial Withdrawal Rate: 4.08%
Real Expected Return (Std. Dev.): 2.0% (11.2%)

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Data Source: Robert Shiller

As the results show, when *long-term* real returns are reduced to just 2%, then *50 percent of all Monte Carlo trials end up being worse than anything that has ever actually happened in history*. In other words, assuming 2% real returns in Monte Carlo analysis may imply there is a 50% probability of a long-term path *worse* than the Great Depression or the stagflationary 1970s! The full comparison at various percentiles is included below.

LOW RETURN MONTE CARLO VS HISTORICAL SCENARIOS
TERMINAL WEALTH AT DIFFERENT PERCENTILES

Percentile	MC Based on Low Return Parameters	Historical Scenarios
0th	Depleted in Year 12	$1
10th	Depleted in Year 20	$365,000
20th	Depleted in Year 22	$545,000
30th	Depleted in Year 25	$685,000
40th	Depleted in Year 27	$875,000
50th	Depleted in Year 30	$1,100,000
60th	$90,000	$1,495,000
70th	$250,000	$1,995,000
80th	$490,000	$2,310,000
90th	$905,000	$2,775,000
100th	$6,605,000	$5,755,000

Assumptions:
Initial Portfolio: $1 million
Initial Withdrawal Rate: 4.08%
Monte Carlo Real Expected Return (Std. Dev.): 5.9% (11.2%)

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Data Source: Robert Shiller

And for those who assume the low end of Blanchett, Finke, and Pfau's spectrum – with real returns at 0% – then Monte Carlo analysis would show that *82% of Monte Carlo trials are worse than anything that has ever happened in US history!*

More than 50% of low-return Monte Carlo scenarios may be worse than all historical sequences!

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Notably, this doesn't mean that the alternative of ignoring today's low yields and high valuation is better. But it is important to understand the full impact of reduced return assumptions in a Monte Carlo analysis, particularly recognizing that *Monte Carlo analysis already projects more long-term tail risk* by not accounting for mean reversion. And, thanks to mean-reversion, it may be the case that even reduced 10-year returns don't necessarily imply dramatic reductions to 30-year returns, though the sequence remains important. Ideally, Monte Carlo analysis tools would allow a combination – such as reduced real returns for 10 years, followed by normalized returns with mean reversion – but, unfortunately, no financial planning software is yet built to provide such regime-based Monte Carlo projections.

The bottom line, though, is simply to recognize that despite the common criticism that Monte Carlo analysis and normal distributions understate "fat tails", when it comes to long-term retirement projections, typical Monte Carlo assumptions actually *overstate extreme outcomes* relative to historical returns due to the failure to account for mean reversion – yielding a material number of projections that are worse (or better) than any sequence that has actually occurred in history. Or, viewed another way, Monte Carlo analysis may actually project *worse returns* than those underlying the 4% safe withdrawal rate research, and choosing a Monte Carlo analysis with a 95% probability of success may unnecessarily constrain a retiree's spending by failing to account for the reality that bear markets do *eventually* fall enough that stocks get cheaper and begin to recover!

Which means, perhaps it's time for financial planning software to do a better job of either allowing advisors to do retirement projections showing actual historical return sequences, or better show what the range of returns really are within a Monte Carlo analysis, in order to help prospective retirees understand just what a 95% Monte Carlo "probability of success" really means!

So what do you think? Do you still believe Monte Carlo analyses understate tail risk… or overstate it? Is it possible that utilizing low return assumptions just overstates tail risk further? How do you set Monte Carlo parameters for your clients? Please share your thoughts in the comments below!